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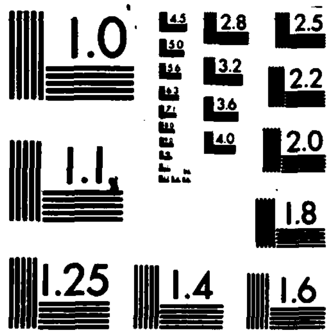

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FOREIGN TECHNOLOGY DIVISION

AN ITERATIVE GUIDANCE METHOD FOR THE LARGE
ROCKET LAUNCH VEHICLE

by

Han Zhuzhai

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AN ITERATIVE GUIDANCE METHOD FOR THE LARGE ROCKET LAUNCH VEHICLE

Han Zhuzhai

ABSTRACT

With the specific objective of solving the rapid control problem for a rocket launch vehicle, this paper introduces and applies an optimum control theory and derives a path adaptive guidance method which satisfies the space guidance requirements--the iterative guidance method. This paper describes the basic concepts and basic conclusion of this method as well as providing a set of approximate guidance formulae. An iterative guidance method is introduced in conjunction with the launching of Earth satellites. The operational procedure of the iterative guidance method and the guidance accuracy obtained from analogue model computation are also presented in this paper. The results of this paper show that the accuracy of the iterative guidance method is much higher than that of the Delta minimum guidance method. It can be concluded that this new method can be widely used in guiding rocket launch vehicles.

I. INTRODUCTION

Before the 1970's, the method for guiding a rocket launch vehicle is based on the Delta minimum method which requires that the interference trajectory of a rocket and the standard trajectory of the rocket be basically on the same geometric trajectory. With the rapid advancements in modern control theory and digital computer technology, scientists have successively applied the path adaptive guidance method in the 1970's for the launching of Earth synchronized satellites and space missions such as the landing on the moon.

The iterative guidance method which is being introduced by this paper here is the most widely used method among the path adaptive methods. Its major characteristic is that the motion of the rocket in the space does not have to follow the prescribed standard trajectory. Based on information provided by the navigational system of the rocket such as the state of the rocket (position, velocity and acceleration) relative to the launch site (or target site), the guidance computer will compute and determine a set of basic commands for the guidance system. In other words, the computer will determine a set of optimum impulse vector directions necessary for the flight mission, hence forming an optimum instantaneous trajectory aiming at the target site.

The iterative guidance method uses two mechanisms to control the rocket; one is to control the impulse direction of the propeller and the other is to determine the time intervals for turning on and turning off the propeller.

This guidance method describes the motion of the rocket by converting the equation of motion for the center of mass of the rocket into the equation of state by introducing the state vector into the equation. This method proposed a control problem for a nonlinear time varying system; with the instantaneous state of the rocket as the initial value, the state of the rocket site as an ultimate constraint, a set of the state angles of the rocket as the control vectors and the minimum flight time between the instantaneous position of the rocket and the target site ξ as a property index. Through the application of the optimum control theory, we can derive a set of necessary conditions for solving the optimum control problem. This is to say that we can obtain a set of maximum value conditions, state equations, accompanying equations and interception conditions with ϕ_ξ (the pitch angle) and ψ_ζ (the off-course angle) as control variables. Here we can call the control variables ϕ_ξ and ψ_ζ the control angles. Theoretically, after the existence of the optimum control solution is confirmed, we can obtain the solution of the optimum

control problem by solving this set of equations. In other words, we can obtain a set of equations representing the optimum control angles ϕ_ξ and ψ_ζ (we call them the guidance equations) as well as the corresponding trajectory. In practical applications, however, the guidance computer in the rocket is not capable of carrying out the complicated computations necessary for accomplishing the above mentioned procedures. This is why some of the parameters in the equations mentioned above will have to be simplified with the prerequisite of not affecting the desirable guidance accuracy and minimum flight time. We can locally approximate the model of the Earth as a plane and proceed with the estimation of the mass and the impulse power of the rocket during future flight times,----. We can subsequently obtain a set of guidance equations suitable for the guidance computer to solve. This paper emphasizes the basic concept and basic conclusion of the iterative guidance method. The derivation of the iterative equations and the detail application of the iterative guidance equations are only described in very general terms.

II. THE PROPOSITION OF THE OPTIMUM CONTROL PROBLEM

Generally speaking, rockets are launched from the ground; they then fly through the atmospheric layer and fly into a vacuum layer. In order to simplify the control methods, the flight of a rocket within the atmospheric layer (altitude lower than 90 km) is usually controlled by fixed procedures under normal circumstances. Guidance control is then added after the rocket enters into the vacuum.

In a 0-$\xi\eta\zeta$ coordinate system, the equation of motion for a rocket flying in the vacuum is:

$$\begin{bmatrix} \ddot{\xi} \\ \ddot{\eta} \\ \ddot{\zeta} \end{bmatrix} = \frac{F}{m}\begin{bmatrix} \cos\varphi_\xi \cos\psi_\zeta \\ \sin\varphi_\xi \cos\psi_\zeta \\ -\sin\psi_\zeta \end{bmatrix} + \begin{bmatrix} g_\xi \\ g_\eta \\ g_\zeta \end{bmatrix} \qquad (1)$$

The $0\text{-}\xi\eta\zeta$ coordinate system is obtained by rotating $(-\beta_c)$ around the oz axis of the gravitational inertial coordinate system 0-xyz at the launchsite.

The 0-xyz coordinate system is a gravitational inertial coordinate system with origin 0 set at the center of the Earth. The oy axis is the vector radius from the center of the Earth to the launchsite, the ox axis is perpendicular to the oy axis and it is pointing toward the target, the oz axis is decided by the right handed system. The 0-xyz coordinate system can be obtained through the platform in the platform-computer scheme; it can also be obtained through a mathematical platform in the continuous scheme.

ϕ_ξ is the angle between the projection of the longitudinal axis of the rocket on the $\xi 0 \eta$ plane and the ξ axis. This is a state angle related to the guidance of a rocket; it is called the pitch angle.

ψ_ζ is the angle between the longitudinal axis of the rocket and the $\xi 0 \eta$ plane. This is another state angle related to the guidance of the rocket and it is called the off-course angle.

g_ξ, g_η and g_ζ are the three components of the function g in the ξ, η and ζ directions. The function g is a very complicated function (gravitational acceleration vector) related to the position of the rocket and the latitude. In order to make the solution of the optimum control problem we are going to propose next easier to solve, we have simplified these values in this paper. We will not discuss the equations representing these functions in any detail here.

With the instantaneous state of the rocket $(\xi_0, \eta_0, \zeta_0, \dot{\xi}_0, \dot{\eta}_0, \dot{\zeta}_0)$ as the initial value, the state of the target site c $(\xi_c, \eta_c, \zeta_c, \dot{\xi}_c, \dot{\eta}_c, \dot{\zeta}_c)$ as the ultimate value, a state vector X can be introduced under these conditions. We will set

$$X=\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}=\begin{bmatrix} \dot{\xi} \\ \xi \\ \dot{\eta} \\ \eta \\ \dot{\zeta} \\ \zeta \end{bmatrix}$$

The equation of motion of the rocket (1) can be expressed by the state equation defined for the time interval $(0,t_e)$ as:

$$\dot{X}=\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix}=\begin{bmatrix} 000000 \\ 100000 \\ 000000 \\ 001000 \\ 000000 \\ 000010 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}+\frac{F}{m}\begin{bmatrix} \cos\varphi_t\cos\psi_t \\ 0 \\ \sin\varphi_t\cos\psi_t \\ 0 \\ -\sin\psi_t \\ 0 \end{bmatrix}+\begin{bmatrix} g_\xi \\ 0 \\ g_\eta \\ 0 \\ g_\zeta \\ 0 \end{bmatrix} \qquad (2)$$

The instantaneous state of the rocket which served as the initial value can be expressed as:

$$X_{t_0}=\begin{bmatrix} x_{10} \\ x_{20} \\ x_{30} \\ x_{40} \\ x_{50} \\ x_{60} \end{bmatrix}=\begin{bmatrix} \dot{\xi}_0 \\ \xi_0 \\ \dot{\eta}_0 \\ \eta_0 \\ \dot{\zeta}_0 \\ \zeta_0 \end{bmatrix} \qquad (3)$$

The state of the target site c which served as the ultimate constraint can be expressed as:

$$X\big|_{t_c}=\begin{bmatrix} x_{1c} \\ x_{2c} \\ x_{3c} \\ x_{4c} \\ x_{5c} \\ x_{6c} \end{bmatrix}=\begin{bmatrix} \dot{\xi}_c \\ \xi_c \\ \dot{\eta}_c \\ \eta_c \\ \dot{\zeta}_c \\ \zeta_c \end{bmatrix} \qquad (4)$$

In equation (2): $\frac{F}{m}=\frac{V_e}{\tau-t}$ is an estimation of the acceleration produced by the impuse power of the propeller at any instant in time during the remainder of the flight trajectory of the rocket (the flight track from the instantaneous position of the rocket to the target site c within the time interval $(0,t_c)$). t is the

flight time within the remainder of the trajectory, with zero second set for the rocket at any instant in time. τ is the instantaneous mass depletion time of the rocket; its value is determined by the ratio between the instantaneous mass of the rocket and the per second fuel consumption rate of the propeller. V_c is the speed of jet propulsion produced by the propeller; it is also called the characteristic speed and its value is determined by the product between the impulse power of the propeller and the weight mass conversion parameter.

x_1, ---, x_6 are state variables

ϕ_ξ, ψ_ζ are control variables.

The gravitational acceleration vector for points along the remainder of the flight trajectory of the rocket is a very complicated function of the unknown position vector. In order to simplify the computation, we have assumed that the Earth model can be locally approximated as a plane for the remainder of the trajectory. At this time, the gravitational acceleration vector within the trajectory can be approximated by the mean value between the instantaneous gravitational acceleration vector at a point and the gravitational acceleration vector at the target site c. This is to say that we have

$$\begin{bmatrix} g_\xi \\ g_\eta \\ g_\zeta \end{bmatrix} = \frac{1}{2}\begin{bmatrix} g_{\xi 0} \\ g_{\eta 0} \\ g_{\zeta 0} \end{bmatrix} + \frac{1}{2}\begin{bmatrix} g_{\xi c} \\ g_{\eta c} \\ g_{\zeta c} \end{bmatrix} \qquad 5)$$

In this equation:

g_ξ, g_η, g_ζ are the three components of the gravitational acceleration vector for each point along the remainder of the trajectory in the $0\text{-}\xi\eta\zeta$ coordinate system.

$g_{\xi 0}, g_{\eta 0}, g_{\zeta 0}$ are the three components of the gravitational acceleration vector for the rocket at an instantaneous point in the $0\text{-}\xi\eta\zeta$ coordinate system.

$g_{\xi c}, g_{\eta c}, g_{\zeta c}$ are the three components of the gravitational acceleration vector for the target site c in the $0\text{-}\xi\eta\zeta$ coordinate system.

The authenticity of the gravitational acceleration represented by the mean gravity increases as the rocket approaches the target. It can be shown from numerical computations that this kind of treatment will not affect the desirable guidance accuracy.

The problem of achieving our desirable goal of guiding the rocket to the target ~~site~~ point c in minimum time is equivalent to the problem of rapid control which requires minimum fuel consumption and maximum effective load for the trajectory since the present rocket propellers are all constant value impulse systems. We can express the property index function as:

$$J=\int_0^{t_c} dt \tag{6}$$

Now, equations (2), (3), (4) and (6) form an optimum control problem for the motion of the rocket which is defined in the time interval $(0, t_c)$.

Solving this optimum control problem is the same as solving for a set of control angles (ϕ_ξ, ψ_ζ) which permits a corresponding rocket flight track that directs the rocket from its initial state X_o to the state of the target site $X|_{t=t_c}$ (the solution of the state equation) within a minimum flight time t_c. This is the optimum instantaneous trajectory of the rocket from any instantaneous point to the target point c.

This optimum control problem is actually a conditional maximum value problem in functional analysis. In order to simplify the solution of this problem, we will temporarily consider the case where there is no end point constraint. This is to say that the final value of the state variable in equation (4) is not fixed.

In order to solve the problem mentioned above, we will introduce the Hamilton function:

$$H=\lambda_1\left(\frac{F}{m}\cos\varphi_\xi\cos\psi_\zeta+g_\xi\right)+\lambda_2x_1+\lambda_3\left(\frac{F}{m}\sin\varphi_\xi\cos\psi_\zeta+g_\eta\right)$$
$$+\lambda_4x_3+\lambda_5\left(-\frac{F}{m}\sin\psi_\zeta+g_\zeta\right)+\lambda_6x_5+1$$

Through mathematical methods, we can obtain a property index J which is equivalent to equation (6). According to the fact that the derivative of the equivalent function J has to be zero when a maximum or a minimum value is reached, we can obtain a set of differential equations and algebraic equations which can be used to solve for the state vectors $(X_1,---X_6)$, the control vectors (ϕ_ξ,ψ_ζ) and the Lagrange multiplication factor $\lambda(\lambda_1,---\lambda_6)$. These equations are:

The state equation and the initial value:

$$\dot{X}=\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\\ \dot{x}_5\\ \dot{x}_6\end{pmatrix}=\begin{pmatrix}000000\\100000\\000000\\001000\\000000\\000010\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\\x_4\\x_5\\x_6\end{pmatrix}+\frac{F}{m}\begin{pmatrix}\cos\varphi_\xi\cos\psi_\zeta\\0\\\cos\varphi_\xi\cos\psi_\zeta\\0\\-\sin\psi_\zeta\\0\end{pmatrix}+\begin{pmatrix}g_\xi\\0\\g_\eta\\0\\g_\zeta\\0\end{pmatrix} \qquad (2)$$

and

$$X_0=\begin{pmatrix}x_{10}\\x_{20}\\x_{30}\\x_{40}\\x_{50}\\x_{60}\end{pmatrix}=\begin{pmatrix}\dot{\xi}_0\\\xi_0\\\dot{\eta}_0\\\eta_0\\\dot{\zeta}_0\\\zeta_0\end{pmatrix} \qquad (3)$$

Maximum or minimum value condition:

$$\begin{bmatrix}\dfrac{\partial H}{\partial\varphi_\xi}\\[2mm]\dfrac{\partial H}{\partial\psi_\zeta}\end{bmatrix}=0$$

this is to say that

$$\begin{bmatrix}\lambda_1\sin\varphi_\xi\cos\psi_\zeta-\lambda_3\cos\varphi_\xi\cos\psi_\zeta\\ \lambda_1\cos\varphi_\xi\sin\psi_\zeta+\lambda_3\sin\varphi_\xi\sin\psi_\zeta+\lambda_5\cos\psi_\zeta\end{bmatrix}=0 \qquad (7)$$

The accompanying equation and the interception are:

$$\dot{\lambda} = -\frac{\partial H}{\partial X} \quad 和 \quad \lambda\delta X|_{t_e} = 0$$

This is to say that

$$\begin{pmatrix} \dot{\lambda}_1 \\ \dot{\lambda}_2 \\ \dot{\lambda}_3 \\ \dot{\lambda}_4 \\ \dot{\lambda}_5 \\ \dot{\lambda}_6 \end{pmatrix} = \begin{pmatrix} -\lambda_2 \\ 0 \\ -\lambda_4 \\ 0 \\ -\lambda_6 \\ 0 \end{pmatrix} \tag{8}$$

and

$$\begin{pmatrix} \lambda_1\delta x_1 \\ \lambda_2\delta x_2 \\ \lambda_3\delta x_3 \\ \lambda_4\delta x_4 \\ \lambda_5\delta x_5 \\ \lambda_6\delta x_6 \end{pmatrix} = 0 \tag{9}$$

By classifying the final values of the state variables prescribed in equation (4) and adding them into the discussion, we can obtain an approximate solution from equations (2), (3), (7), (8) and (9); the control angles ϕ_ξ and ψ_ζ can be represented by the equation:

$$\begin{cases} \varphi_\xi = \bar{\varphi}_\xi + k_2 t - k_1 \\ \psi_\zeta = \bar{\psi}_\zeta + e_2 t - e_1 \end{cases} \tag{10}$$

In this equation:

$\bar{\varphi}_\xi, \bar{\psi}_\zeta$ are the control angles which satisfy the velocity vector at the target point; k_1, k_2, e_1 and e_2 are the additional control angular parameters produced by the system so that the positional vector of the target point can be reached.

Since it is realatively difficult to solve for the approximate solution using the equations mentioned above, we will proceed with

a rough introduction of this problem in the "solution of the optimum control problem" section of this paper.

III. SOLUTION OF THE OPTIMUM CONTROL PROBLEM

We have already defined the $0\text{-}\xi\eta\zeta$ coordinate system as a system obtained from rotating the instantaneous gravitational inertial coordinate system at the launch site of the rocket 0-xyz around the 0z axis by $(-\beta_c)$. The transformation relationship between the two coordinate systems is thus:

$$\begin{bmatrix} \xi \\ \eta \\ \zeta \end{bmatrix} = \begin{bmatrix} \cos\beta_c & -\sin\beta_c & 0 \\ \sin\beta_c & -\cos\beta_c & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{11}$$

In this equation:

β_c is the projection of the total navigational angle between the launch site of the rocket and the target point c on the xoy plane. It is a combination of two separate parts. The first part is a projection of the navigational angle between the launch site of the rocket and an instantaneous point on the xoy plane β_c, the second part is a projection of the navigational angle for the remainder of the trajectory between the instantaneous point and the target point c on the xoy plane β_t.

β_c can be computed by using the instantaneous rocket position x,y provided b y the navigational computation. β_t can be obtained approximately by using the local horizontal component of the projection of the instantaneous rocket velocity vector on the xoy plane as the initial speed and the local horizontal component of the projection of the acceleration vector produced by the impulse power of the propeller on the xoy plane as the acceleration. Note that the acceleration is estimated locally at the target point c. The authenticity of the flight path represented by this approximation increases as the rocket approaches the target point.

The equations representing the specific estimation of β_c are:

$$\beta_c = \beta_e + \beta_t$$

$$\beta_e = \sin^{-1} \frac{x}{(x^2 + y^2)^{1/2}}$$

$$\beta_t = \frac{1}{\eta_e} \Big\{ V^* t_e \cos \theta_H^* + V_e \cos \theta_{HC} \times \left[(\tau_{\xi\eta} - t_e) \ln \frac{\tau_{\xi\eta} - t_e}{\tau_{\xi\eta}} + t_e \right] \Big\}$$

$$V^* = (\dot{x}^2 + \dot{y}^2)^{1/2}$$

$$\tau_{\xi\eta} = \frac{Ve}{(\dot{W}_x^2 + \dot{W}_y^2)^{1/2}}$$

$$\cos \theta_H^* = \frac{|x\dot{y} - y\dot{x}|}{V^*(x^2 + y^2)^{1/2}}$$

$\cos \theta_{HC}$ is a given constant; it represents the cosine of the inclination angle of the trajectory at the target point.

$\dot{W}_x, \dot{W}_y, \dot{W}_z$ are the three components of the apparent acceleration of the rocket $\dot{W}$ measured by sensitive instruments in the 0-xyz coordinate presystem.

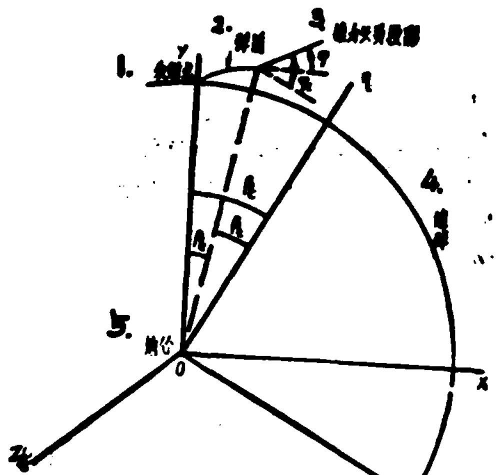

Figure 1. Coordinate system for the iterative guidance method.
1--launch point; 2--trajectory; 3--projection of the impulse vector; 4--Earth; 5--center of the Earth

ϕ, ψ are the pitch angle and the off-course angle of the rocket in the 0-xyz coordinate system, respectively. It is obvious that the pitch angle ϕ_ξ and the off-course angle ψ_ζ are related to ϕ and ψ as follows:

$$\varphi_\xi = \varphi + \beta_0$$
$$\psi_\zeta = \psi$$

From equation (7), which specifies the maximum value conditions, we can obtain the equations representing the control variables ϕ_ξ and ψ_ζ. The equations are:

$$\begin{cases} \varphi_\xi = tg^{-1}\left(\dfrac{\lambda_3}{\lambda_1}\right) \\ \psi_\zeta = tg^{-1}\left(-\dfrac{\lambda_5}{\lambda_1}\cos\varphi_\xi\right) \end{cases} \tag{12}$$

By integrating the accompanying equation (8), we can obtain

$$\begin{cases} \lambda_1 = \lambda_{10} - \lambda_{20} t \\ \lambda_3 = \lambda_{30} - \lambda_{40} t \\ \lambda_5 = \lambda_{50} - \lambda_{60} t \end{cases} \tag{13}$$

In this equation, the λ_{io} are integration constants with i = 1,---6.

It is obvious that the representative equations of the control angles ϕ_ξ and ψ_ζ depend on the determination of the integration constants λ_{io}. We will now classify the final values of the state variables (the state of the target point c) prescribed by equation (4) and add them into our discussion. We expect to obtain a set of specific representative equations for the control angles ϕ_ξ and ψ_ζ.

(i) Assuming that the velocity vector $(\dot{\xi}_c, \dot{\eta}_c, \dot{\zeta}_c)$ at the target point is a fixed value, while the position vector (ξ_c, η_c, ζ_c) is not fixed.

Since (ξ_c, η_c, ζ_c) is not fixed, the final value of the corresponding state variables x_2, x_4, x_6 are also not fixed. This is

the same as saying $\Delta x_i)_{t_0} \neq 0$ $(i=2,4,6)$. We can then obtain the following equation from the interception condition equation (9):

$$\lambda_{20}=\lambda_{40}=\lambda_{60}=0$$

We should consider that equation (13) becomes:

$$\begin{cases} \lambda_1=\lambda_{10} \\ \lambda_3=\lambda_{30} \\ \lambda_5=\lambda_{50} \end{cases}$$

At this time equation (12) becomes:

$$\begin{cases} \varphi_t=\mathrm{tg}^{-1}\dfrac{\lambda_{30}}{\lambda_{10}} \\ \psi_t=\mathrm{tg}^{-1}\left(-\dfrac{\lambda_{50}}{\lambda_{10}}\cos\varphi_t\right) \end{cases} \tag{12*}$$

Equation (12*) indicates the instantaneous control angles should be a set of instantaneous constants if the rocket is required to reach the pre-selected velocity vector at the target point c. We might as well indicate them as:

$$\begin{cases} \varphi_t=\bar{\varphi}_t \\ \psi_t=\bar{\psi}_t \end{cases} \tag{14}$$

Since we can decompose the spatial motion of the rocket into a combination of horizontal motion and lateral motion, we can rewrite the equation of motion of the rocket (1) by incorporating the decomposition with the characteristics of inertial guidance and obtain another equation as:

$$\begin{pmatrix} \ddot{\xi} \\ \ddot{\eta} \\ \ddot{\zeta} \end{pmatrix} = \begin{pmatrix} \left(\dfrac{F}{m}\right)_{\xi\eta}\cos\varphi_t \\ \left(\dfrac{F}{m}\right)_{\xi\eta}\sin\varphi_t \\ -\dfrac{F}{m}\sin\psi_t \end{pmatrix} + \begin{pmatrix} g_\xi \\ g_\eta \\ g_\zeta \end{pmatrix} \tag{1*}$$

In this equation:

$$\left(\frac{F}{m}\right)_{\xi\eta}=\frac{F}{m}\cos\psi_t=\frac{V_e}{\tau_{\xi\eta}-t}$$

$$\frac{F}{m}=\frac{V_e}{\tau-t}$$

$$\tau_{\xi\eta}=\frac{V_e}{W^*}$$

$$\tau=\frac{V_e}{W}$$

$$W^*=(W_x^2+W_y^2)^{1/2}$$
$$W=(W_x^2+W_y^2+W_z^2)^{1/2}$$

We now substitute equation (14) into equation (1*) within the time interval $(0, t_c)$ and seek the solutions by integrating the equations and setting the initial value

$(\xi, \dot{\xi}, \eta, \dot{\eta}, \zeta, \dot{\zeta})|_0=(\xi_0, \dot{\xi}_0, \eta_0, \dot{\eta}_0, \zeta_0, \dot{\zeta}^0)$ and the final value

$(\dot{\xi}, \dot{\eta}, \dot{\zeta})|_{t_c}=(\dot{\xi}_c, \dot{\eta}_c, \dot{\zeta}_c)$. We can obtain

$$\begin{cases} \varphi_\xi = \mathrm{tg}^{-1}\dfrac{\dot{\eta}_c-\dot{\eta}_0-g_\eta t_c}{\dot{\xi}_c-\dot{\xi}_0-g_\xi t_c} \\ \psi_\zeta = \sin^{-1}\dfrac{\dot{\zeta}_c-\dot{\zeta}_0+g_\zeta t_c}{B_1} \end{cases} \tag{15}$$

In this equation:

$$B_1=V_e\ln\frac{\tau}{\tau-t_c}$$

The remaining flight time t_c is a time estimate for the rocket to complete the remainder of its trajectory, with t_c equals zero second at the particular instant in time when this estimation is carried out.

The computation of t_c is based on the control requirement. We can obtain the equations necessary for solving t_c by establishing the concept that the expected velocity increment between the instantaneous position of the rocket and the target point c must be equal to the residual velocity increment computed by integrating the equation of motion (1) on the time interval $(0, t_c)$. These equations are:

$$\begin{cases} \Delta V^2 = (\dot{\xi}_c - \dot{\xi}_0 - g_\xi t_c)^2 + (\dot{\eta}_c - \dot{\eta}_0 - g_\eta t_c)^2 + (\dot{\zeta}_c - \dot{\zeta}_0 - g_\zeta t_c)^2 \\ t_c = \tau\left(1 - e^{-\frac{\Delta V}{V_e}}\right) \end{cases}$$

These equations can be used by the guidance computer; t_c values which satisfy our occuracy requirements can be obtained through iterative computations.

(ii) Assuming that the velocity vector $(\dot{\xi}_c, \dot{\eta}_c, \dot{\zeta}_c)$ and two position components η_c and ζ_c are determined at the target point c, ξ is not fixed.

It can be shown from the principles of rocket guidance that the control angles $\bar{\phi}_\xi, \bar{\psi}_\zeta$, necessary to insure that the rocket reaches its preselected velocity vector at the target point c are the major constituent parts of the control angles ϕ_ξ and ψ_ζ while the additional control angles necessary for the formation of the position vector are the minor constituent parts. We might as well assume that:

$$\begin{cases} \varphi_\xi = \bar{\phi}_\xi + k_2 t - k_1 \\ \psi_\zeta = \bar{\psi}_\zeta + e_2 t - e_1 \end{cases} \qquad (16)$$

By treating $(k_2 t - k_1), (e_2 t - e_1)$ as small quantities, expanding $\cos\varphi_\xi, \sin\psi_\zeta, \cos\psi_\zeta$ approximately and keeping only the first order terms and substituting these terms into equation (1*) at time interval 0, t_c we carry out integral solution according to initial value $(\dot{\xi}, \xi, \dot{\eta}, \eta, \dot{\zeta}, \zeta)|_0 = (\dot{\xi}_0, \xi_0, \dot{\eta}_0, \eta_0, \dot{\zeta}_0, \zeta_0)$ and terminal value $(\dot{\xi}, \dot{\eta}, \eta, \dot{\zeta}, \zeta)|_{t_c} = (\dot{\xi}_c, \dot{\eta}_c, \eta_c, \dot{\zeta}_c, \zeta_c)$ and we can obtain the computational equations for k_1, k_2, e_1, e_2 . We will present the computations of these representative equations in "Guidance equations and explanation of their applications".

(iii) The velocity vector $(\dot{\xi}_c, \dot{\eta}_c, \dot{\zeta}_c)$ and two position components ξ_c and ζ_c are determined at the target point c; η_c is not fixed.

We can obtain a set of equations for the computation of k_1, k_2, e_1 and e_2 by going through procedures similar to those in (ii).

The optimum control angle obtained for such an optimum control problem, however, will guide the rocket to fly towards the crust of the Earth in an attempt to guide the rocket to reach the target within a minimum time. This does not satisfy the permissable rocket flight track under practical conditions so we will no longer discuss this result in this paper.

If we wanted to obtain a feasible conclusion, we have to consider adding a constraint on the vector radius $r=(\xi^2+\eta^2+\zeta^2)^{1/2}$ during the flight of the rocket. By solving for the rapid control problem of this kind of a system, we can realize the optimum control for this type of final constraint.

(iv) Both the velocity vector and the position vector are fixed at the target point c.

For this type of requirement, the rocket will have to simultaneously satisfy the final constraint of three velocity components and three position components. Intuitively, this requirement for optimum control cannot be realized by rockets which can only be adjusted to change the direction of the impulse power generated by the propeller, not the magnitude of the impulse power generated by the propeller. Theoretically, this involves the research into whether or not this type of a system can be controlled. The numerical computation shows, however, that satisfactory guidance accuracy can be achieved by guiding the rocket using the guiding equations described in (ii). It can be shown that those equations will not produce an excessively large deviation in the position component (ξ_c).

Depending on the special requirements of certain rocket launch vehicles, we can also interchange the k_1, k_2, e_1 and e_2 values obtained separately from conditions (ii) and (iii) so that the required target ranged can be reached. It should be noted that whenever the rocket is being guided by (iii), the requirement

that the rocket flight vector radius $r=(\xi^2+\eta^2+\zeta^2)^{1/2}$ be greater than the minimum constraining radius r_{**} must be satisfied so that we can insure that the rocket will not collide with the crust of the Earth. If this requirement cannot be fulfilled, we should guide the rocket according to (ii).

IV. THE GUIDING EQUATIONS AND EXPLANATION FOR THEIR APPLICATION

In order to facilitate the computation of the guidance computer, we have summed up the computational parameters and the guiding equations and arranged in the following format:

(i) The parameters used for the guidance computations in each level: $V_e, g_{\xi e}, g_{\eta e}, g_{\zeta e}, \cos\theta_{Hc}, \dot{\xi}_e, \dot{\eta}_e, \dot{\zeta}_e, \xi_e, \zeta_e, \eta_e, \Delta t, \varepsilon, t_e^{(0)}$ (for application in the first order guidance computations in each level).

(ii) The instantaneous measured values and the values obtained from navigational computations which are used at each time point throughout the guidance computations:

$$\dot{W}_x, \dot{W}_y, \dot{W}_z, \dot{x}, \dot{y}, \dot{z}, x, y, z, g_x, g_y, g_z$$

(iii) The computational guiding equations:

(1) To obtain the control angle ϕ:

$$(a)\begin{cases} r^* = (x^2+y^2)^{1/2} \\ \beta_e = \sin^{-1}\left(\dfrac{x}{r^*}\right) \\ V^* = (\dot{x}^2+\dot{y}^2)^{1/2} \\ \cos\theta_H^* = \dfrac{|x\dot{y}-y\dot{x}|}{r^* V^*} \end{cases}$$

$$(b)\begin{cases} \dot{W}^* = (\dot{W}_x^2+\dot{W}_y^2)^{1/2} \\ \dot{W} = (\dot{W}_x^2+\dot{W}_y^2+\dot{W}_z^2)^{1/2} \\ \tau = \dfrac{V_e}{\dot{W}} \\ \tau_{\xi\eta} = \dfrac{V_e}{\dot{W}^*} \end{cases}$$

$$
(c)\begin{cases}
A_1 = V_e \ln \dfrac{\tau_{\xi e}}{\tau_{\xi e} - t_e} \\
A_2 = A_1 \tau_{\xi e} - V_e t_e \\
A_3 = A_1 t_e - A_2 \\
A_4 = A \tau_{\xi e} - \dfrac{1}{2} V_e t_e^2
\end{cases}
$$

$$
(d)\begin{cases}
\beta_t = \dfrac{1}{\eta_e}\left(V^* t_e \cos \theta_H^* + A_4 \cos \theta_{HC}\right) \\
\beta_e = \beta_o + \beta_t
\end{cases}
$$

$$
(e)\begin{cases}
\begin{bmatrix} \xi \\ \eta \\ \zeta \end{bmatrix} = \begin{bmatrix} \cos\beta_e & -\sin\beta_e & 0 \\ \sin\beta_e & \cos\beta_e & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \\
\begin{bmatrix} \dot\xi \\ \dot\eta \\ \dot\zeta \end{bmatrix} = \begin{bmatrix} \cos\beta_e & -\sin\beta_e & 0 \\ \sin\beta_e & \cos\beta_e & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot x \\ \dot y \\ \dot z \end{bmatrix} \\
\begin{bmatrix} g_\xi \\ g_\eta \\ g_\zeta \end{bmatrix} = \dfrac{1}{2} \begin{bmatrix} \cos\beta_e & -\sin\beta_e & 0 \\ \sin\beta_e & \cos\beta_e & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix} + \dfrac{1}{2} \begin{bmatrix} g_{\xi e} \\ g_{\eta e} \\ g_{\zeta e} \end{bmatrix}
\end{cases}
$$

$$
(f)\begin{cases}
\Delta V^2 = (\dot\xi_e - \dot\xi - g_\xi t_e)^2 + (\dot\eta_e - \dot\eta - g_\eta t_e)^2 + (\dot\zeta_e - \dot\zeta - g_\zeta t_e)^2 \\
t_e = \tau\left(1 - e^{-\frac{\Delta V}{V_e}}\right)
\end{cases}
$$

$$
(g)\begin{cases}
\phi = \mathrm{tg}^{-1}\left[\dfrac{\dot\eta_e - \dot\eta - g_\eta t_e}{\dot\xi_e - \dot\xi - g_\xi t_e}\right] \\
P = A_3 \cos \phi_\xi \\
Q = A_4 \cos \phi_\xi \\
R = \eta_e - \eta - \dot\eta t_e - \dfrac{1}{2} g_\eta t_e^2 - A_3 \sin \phi_\xi \\
\Delta k = A_1 Q - A_2 P \\
k_1 = \dfrac{A_2 R}{\Delta k} \\
k_2 = \dfrac{A_1 R}{\Delta k} \\
\varphi_\xi = \phi_\xi + k_2 \Delta t - k_1 \\
\varphi = \varphi_\xi - \beta_e
\end{cases}
$$

(2) To obtain the control angle ψ:

$$B_1 = V_e \ln \frac{\tau}{\tau - t_c}$$

$$B_2 = B_1 \tau - V_e t_c$$

$$B_3 = B_1 t_c - B_2$$

$$B_4 = B_3 \tau - \frac{1}{2} V_e t_c^2$$

$$\bar{\phi}_\zeta = \sin^{-1}\left[\frac{\dot{\zeta} - \dot{\zeta}_c + g_\zeta t_c}{B_1}\right]$$

$$E = B_2 \cos \bar{\phi}_\zeta$$

$$G = B_4 \cos \bar{\phi}_\zeta$$

$$H = \zeta_c - \zeta - \dot{\zeta} t_c - \frac{1}{2} g_\zeta t_c^2 + B_3 \sin \bar{\phi}_\zeta$$

$$\Delta_e = B_2 E - B_1 G$$

$$e_1 = \frac{B_2 H}{\Delta e}$$

$$e_2 = \frac{B_1 H}{\Delta e}$$

$$\psi = \psi_\zeta = \bar{\phi}_\zeta + e_2 \Delta t - e_1$$

The specific computational procedure of the guidance computer is to first take the pre-estimated residual flight time of the rocket $t_c^{(o)}$ as the initial value during the first computation of the guiding equation at all levels. After several iterations of ΔV and t_c, we will take the $t_c^{(n)}$ value which satisfies $|t_c^{(n)} - t_c^{(n-1)}| < \varepsilon$ (if necessary, we can also go through several cycles from equations (c) to (f)), and continue the computation with the right sequence. Eventually, we will feed the computational results ϕ and ψ into the state control loop, control the propeller to deflect the impulse power to the optimum direction and complete the guidance procedure at the first time point. During the entire flight time, the latter time point always uses the computational result t_c of a prior time point minus the time interval Δt as its initial value $t_c^{(o)}$ (same as $t_c - \Delta t$). By repeating the computational procedures mentioned above, the rocket will ultimately be guided to the end point and all pre-selected states of the target point will be satisfied.

Based on the guidance principles, the optimum control is generally added to the system after the rocket has flown through the atmospheric layer. This procedure will insure the stability and the reliability of the aviation system of the rocket. In addition to this, the time in which the guidance mechanism is initiated or terminated cannot be the same as the time when the propeller is turned on or off. In other words, the optimum guidance is introduced after the propeller has been started, and the optimum guidance is terminated before the propeller is turned off.

On the other hand, the $R=\eta_c-\eta-\dot{\eta}t_c-\frac{1}{2}g_\eta t_c^2-\dot{A}_c\sin\dot{\psi}_t$ represents a deviation value between the estimated position along the η direction and the pre-selected position of the target point c. The estimation of the position is based on a continuous flight track towards the target point under the action of the instantaneous velocity vector of the rocket and the acceleration vector produced by the impulse power of the propeller. This deviation value consists of two parts. One part is the position difference quantity $(\eta_c-\eta)$, and the other part is a position quantity produced by the instantaneous velocity, the impulse acceleration of the propeller and the component of the gravitational acceleration. This second part is produced in an attempt to compensate for the position difference quantity. The results of numerical computations indicate that when the rocket approaches the target, the numerical value of the position difference is far less than the numerical value of the other terms combined in the expression for R. At this time, the k_1 and k_2 produced by R will generate an excessive additional part $(k_2\Delta t-k_1)$ in the control angle ϕ. Similarly, the H in the equation will also generate an excessive additional part $(e_2\Delta t-e_1)$ in the control angle ψ.

In order to avoid destroying the stability of the aviation system of the rocket while still maintaining a sufficiently high degree of accuracy, we have applied the method of setting k_1, k_2, e_1 and e_2 to be zero under separate conditions in advance. For

the numerical computation of a certain multistage rocket which sent a satellite into the Earth's orbit, the specific method is:

$$
\begin{array}{ll}
t_e < T_1 = 60'' & \epsilon_1 = \epsilon_2 = 0 \\
t_e < T_2 = 40'' & k_1 = k_2 = 0 \\
t_e < T_3 = 20'' & \psi = \psi(20'') \\
t_e < T_4 = 10'' & \varphi = \varphi(10'')
\end{array}
\Bigg\}\ \text{until the propeller is turned off}
$$

V. GUIDANCE METHOD AND ITS FLOW CHART

The guidance scheme for the accomplishment of a space flight mission of a rocket (such as the launching of an Earth satellite) is generally consisted of navigation, guidance and the control of the on-off of the propeller.

The mission of the navigational system is to use sensitive instruments to measure the apparent velocity and the apparent acceleration of the rocket itself and subsequently obtain a set of instantaneous position, velocity and gravitational acceleration of the rocket after the measured information has been transmitted to and processed by the guidance computer. The function of the guidance system is to take the information obtained by the navigational system, the results of the computations and certain related quantities which have already been entered into the system and subsequently compute the guiding equations through the guidance computer. The guidance system then predicts the optimum impulse direction of the propeller at any instant in time so that the rocket can reach the target accurately. When the actual flying velocity of the rocket $V(V^2=\dot{x}^2+\dot{y}^2+\dot{z}^2)$ equals the prescribed velocity $V_e(V_e=\xi_e^2+\eta_e^2+\zeta^2)$ (or use $t_c = 0$), the guidance will issue a command to shut off the propeller, hence completing its control concerning the turning off of the propeller.

The operational procedures of the iterative guidance scheme are shown in Figure 2.

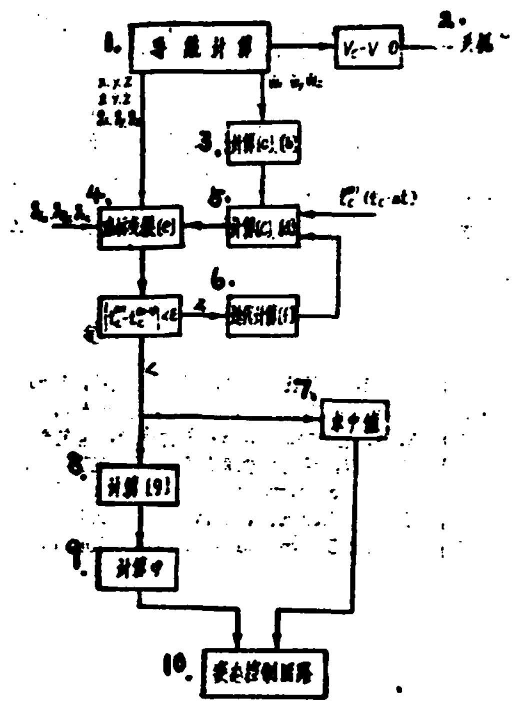

Figure 2. The computational procedure of the iterative guidance scheme.
1--navigational computation; 2--shutting off the propeller; 3--computation; 4--transformation of coordinate system; 5--computation; 6--iterative computation; 7--to obtain ψ value; 8--computation; 9--compute ϕ; 10--state control loop

VI. CONCLUSION

Based on the instantaneous state of the rocket and the state of the pre-selected target point, the iterative guidance method predicts the future motion principles of the rocket, forms mobile aerial flight commands for the rocket and directs the rocket towards the target. Since a minimum flight time is the basis of the iterative guidance method, the impulse direction of the rocket propeller determined by this method has the effect of fuel saving and the effective load of the trajectory is also increased.

The numerical computation of the iterative guidance scheme for launching a certain Earth satellite shows that as far as the guidance accuracy of the point in which the satellite entered the

1. 误差量 5 项目 / 2. 方案	ΔV m/s	Δr km	6 $\Delta\theta_H$ 度	Δa km	Δr_p km	Δr_a km	7 ΔI 度	8 $\Delta\Omega$ 度
3. 迭代制导方案	0.100	0.010	0.00048	3.2214	0.013	4.541	0.00458	0.0014
4. delta制导方案	4.516	3.6537	0.046	76.894	3.650	155.416	0.04548	0.072219

Δ: represents deviation
V: velocity of the rocket
r: vector radius of the rocket
θ_H: inclination angle of the rocket at the end point of the trajectory
a: major axis of the satellite orbit
r_p: vector radius of the satellite orbit at a point closest to Earth
r_a: vector radius of the satellite orbit at a point farthest away from Earth
I: inclination angle of the satellite
Ω: focus of the satellite orbit

1--deviation quantity; 2--method; 3--iterative guidance scheme; 4--delta guidance method; 5--category; 6--$\Delta\theta_H$ degree; 7--ΔI degree; 8--ΔΩ degree

orbit is concerned, the accuracy of the iterative guidance method is generally 10 times higher than that of the delta method. The following table lists the various maximum deviations in the orbit entry point associated with the iterative guidance method and the guidance method.

When the delta guidance method is applied to a rocket generating system with multiple propellers, the failure of one single propeller will cause the failure of the entire flight operation. When the iterative guidance system is used, however, the mission will be fulfilled even if this kind of a situation occurred.

In addition, the iterative guidance method does not require repeated adjustment of the parameters in the guiding operations and the massive computation of the interference trajectories since the iterative guidance method does not require that the interference trajectory and the standard trajectory be basically

overlapped. Only a small number of interference trajectories are computed for the observation of the guidance accuracy, hence drastically reduce the ground design and computational work load. For different types and different launch missions, all the iterative guidance method requires is the transmission of the fixed variables of the rocket and the state of the target into the guidance computer and the proper selection of the time quantities T_1,---,T_4. A concrete guidance scheme can then be obtained based on this information. This is why the iterative guidance methods also possess certain advantages such as design, flexibility, short design time and ease of performing mobile launching missions.

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